Integrable Ring Design

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Integrable Particle Dynamics in Accelerators
January 2019 USPAS at Knoxville

Design Challenges

Applications at Fermilab

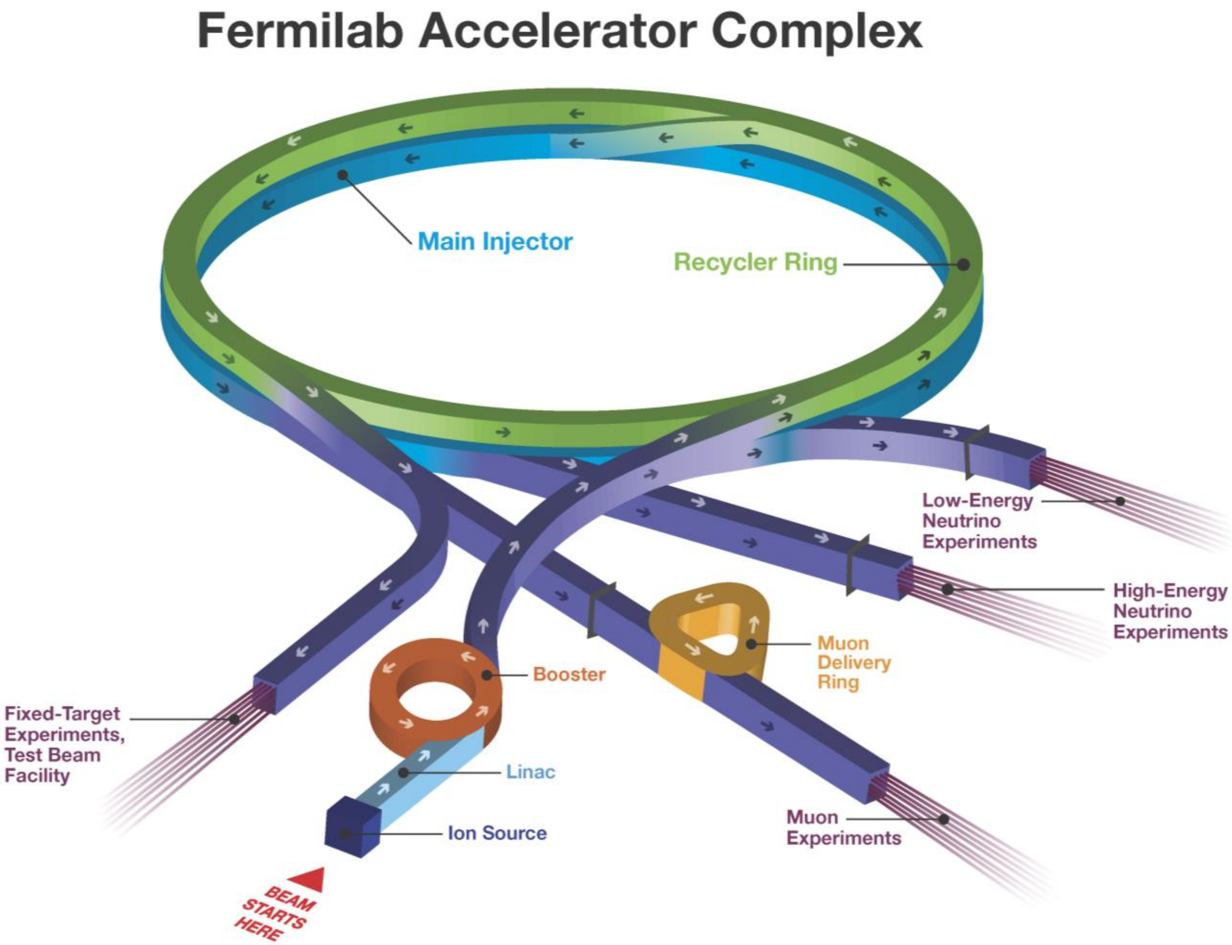
What goes into an integrable lattice design?

What happens when there is chromaticity & momentum spread?

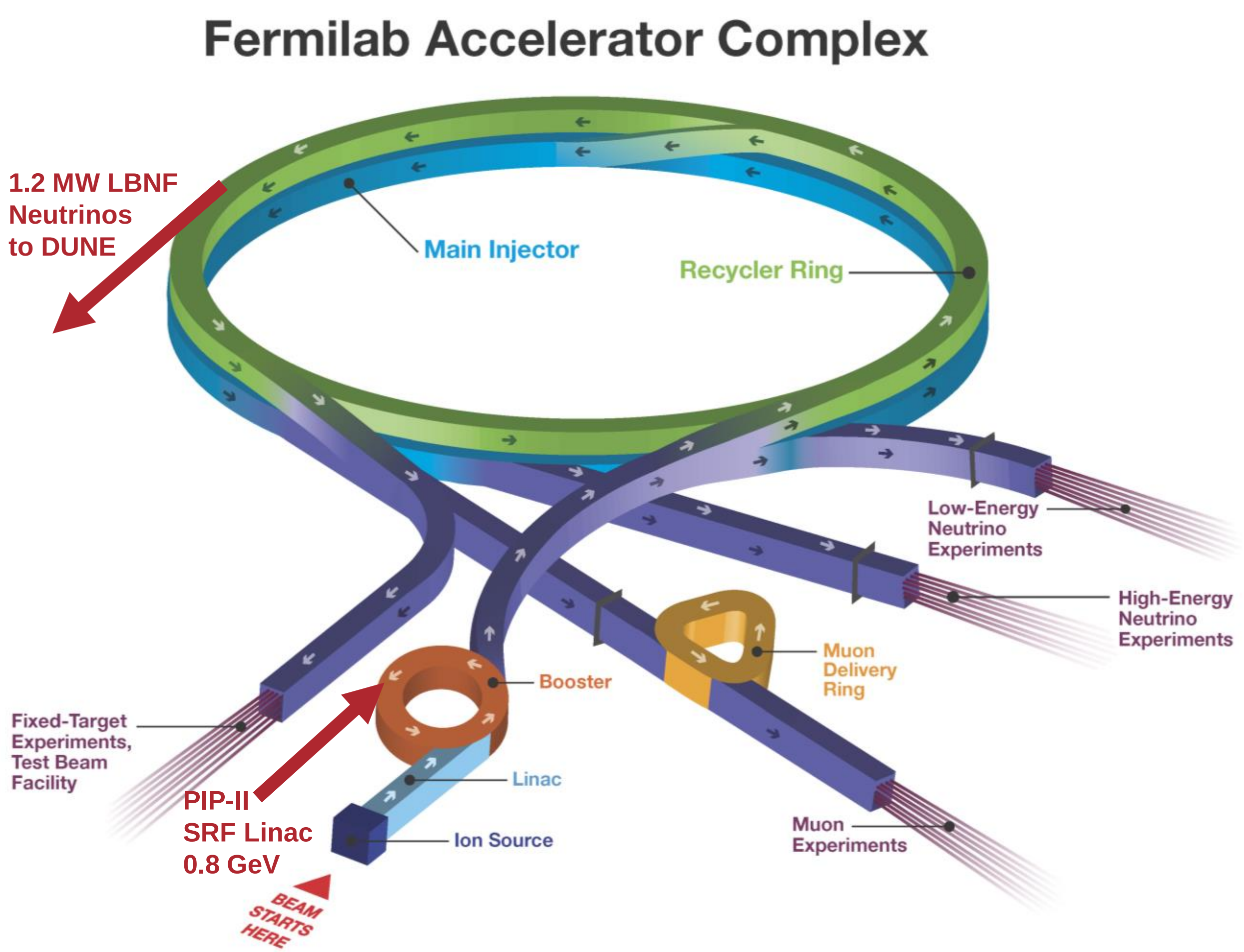
What happens when there is space-charge forces?

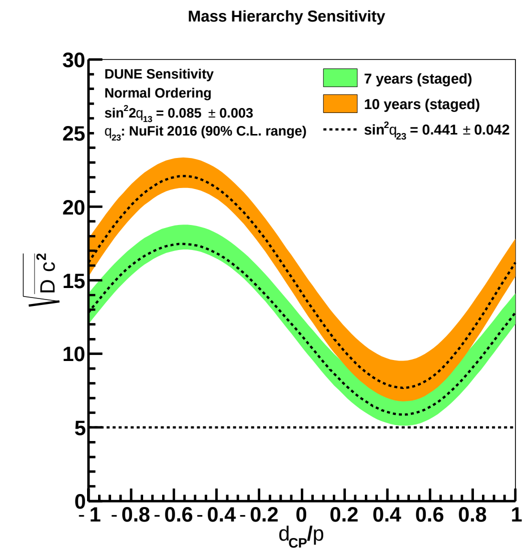
Motivation

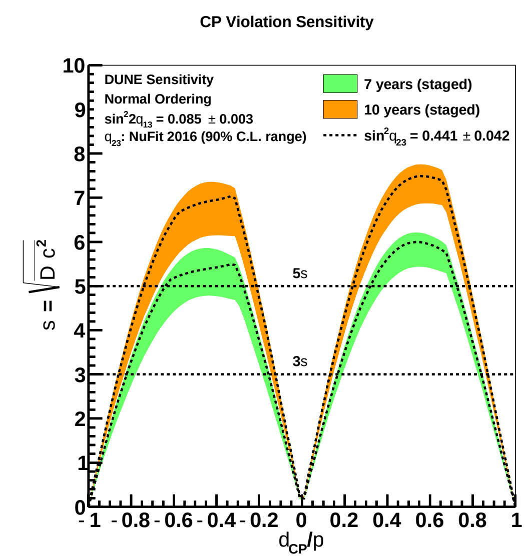
Fermilab Upcoming Upgrades



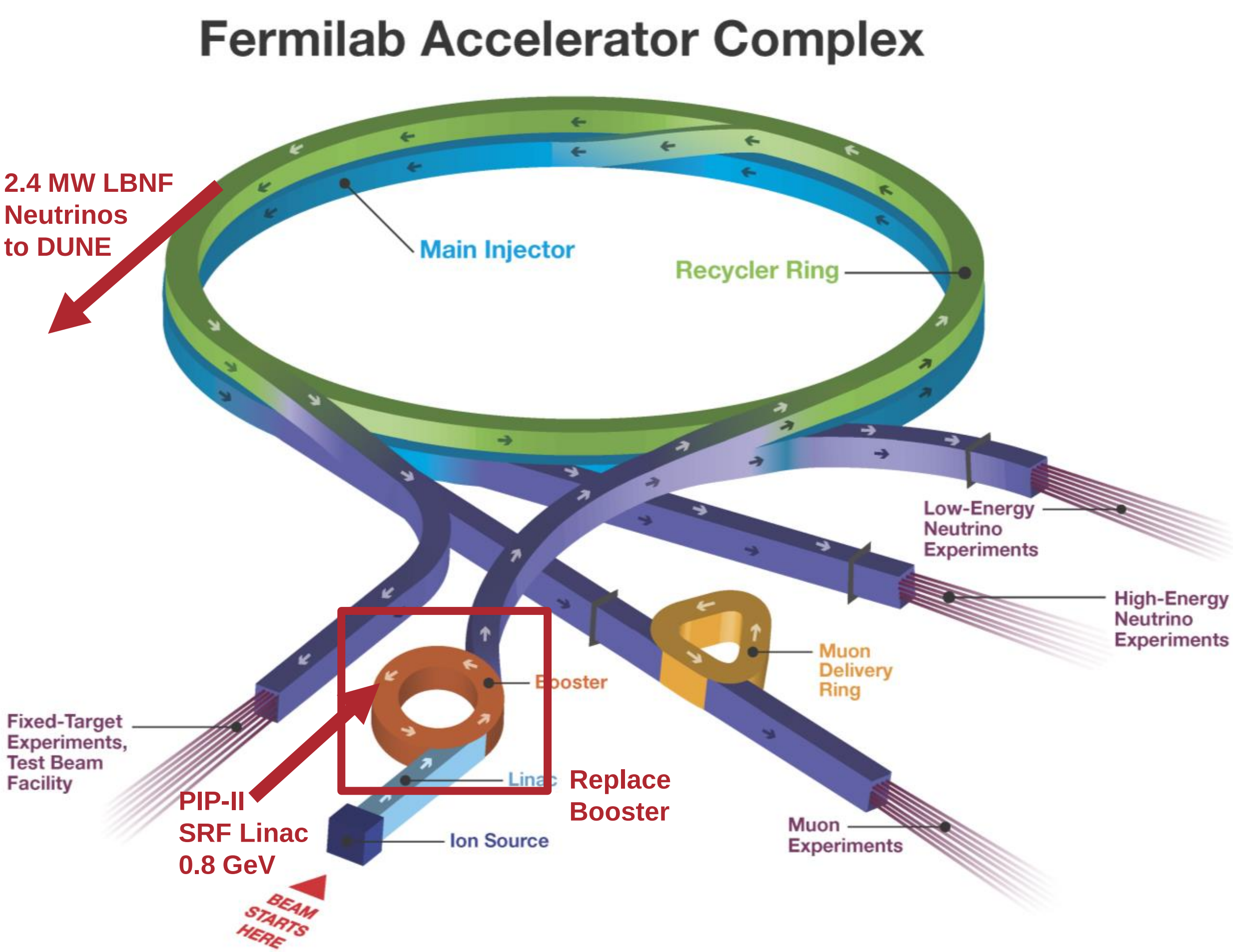
Fermilab Upcoming Upgrades 1.2MW, ~2026

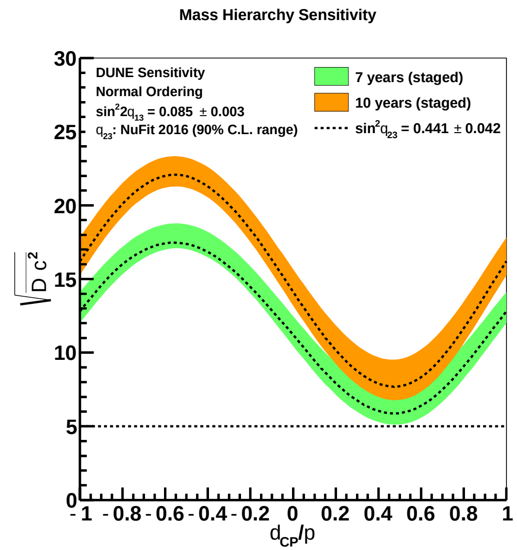


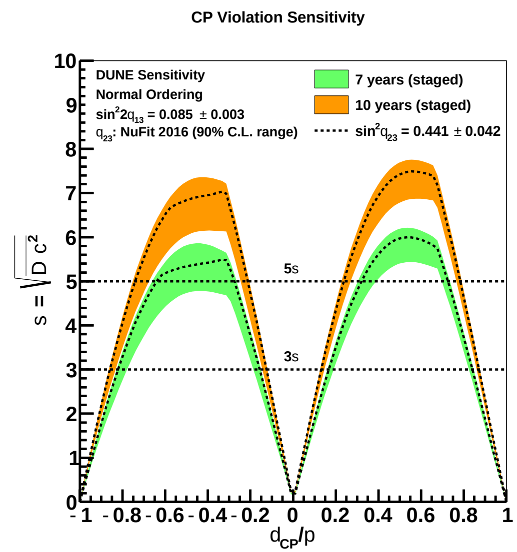




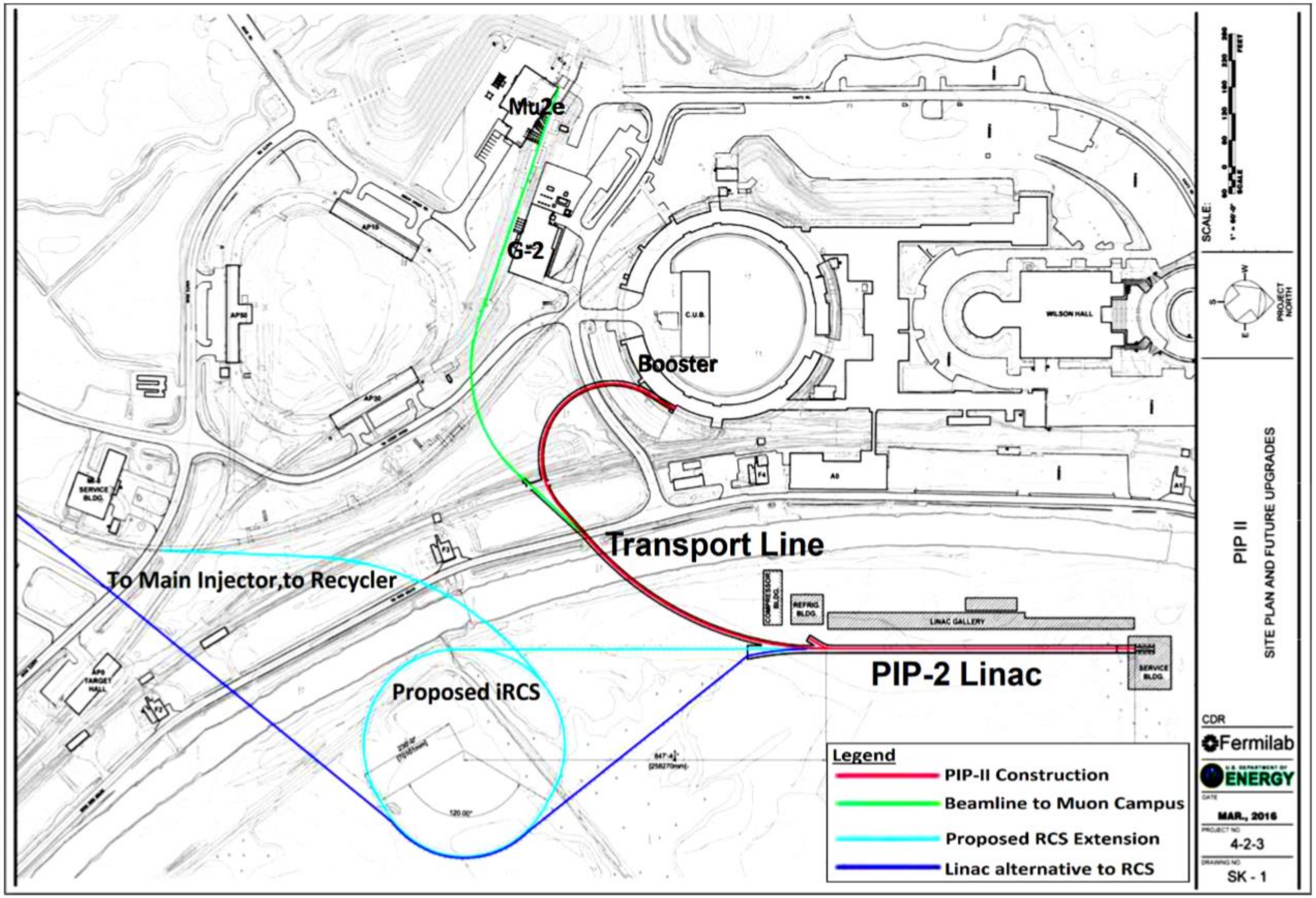
Fermilab Upcoming Upgrades 2.4MW, ~2032



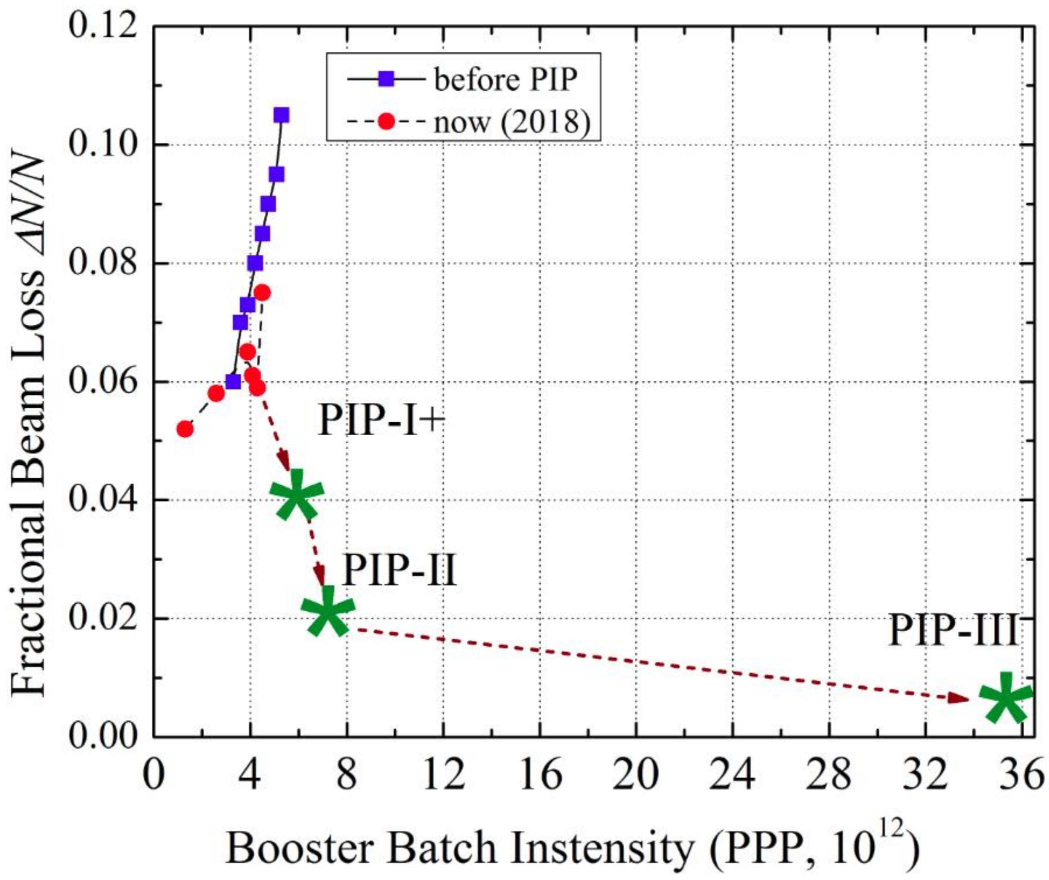




"PIP-III" New RCS to replace Booster



Fermilab Loss Limits



Shiltsev

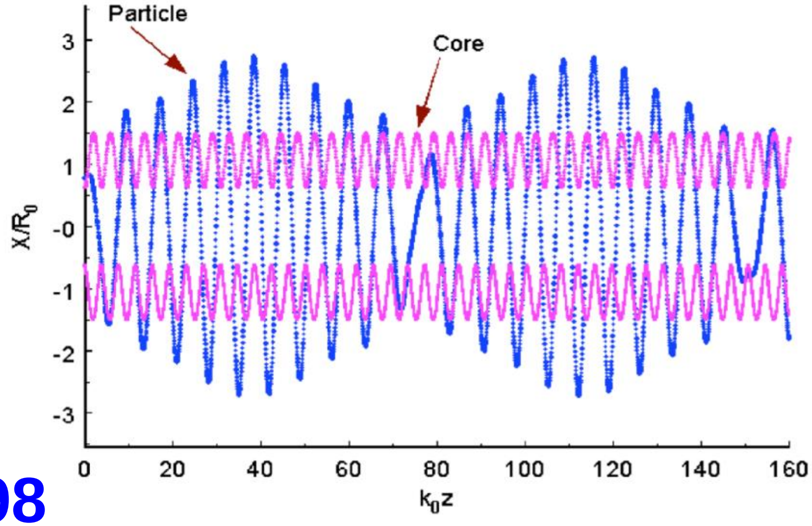
Radio-activation of particle accelerator a major operation limits. Losses must be kept within absolute limits, which mean power increases require a reduction in *loss rate*.

Core-Halo Model

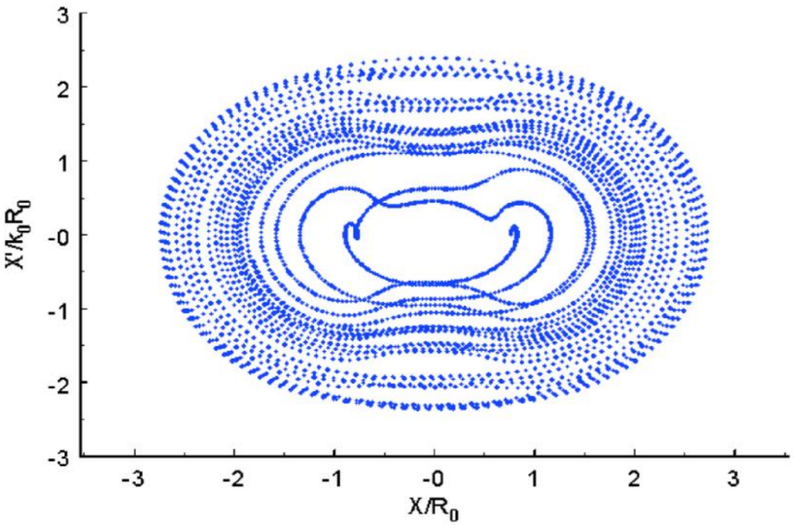
$$\frac{d^2R}{dz^2} + k_0^2 R - \frac{\varepsilon^2}{R^3} - \frac{K}{R} = 0$$

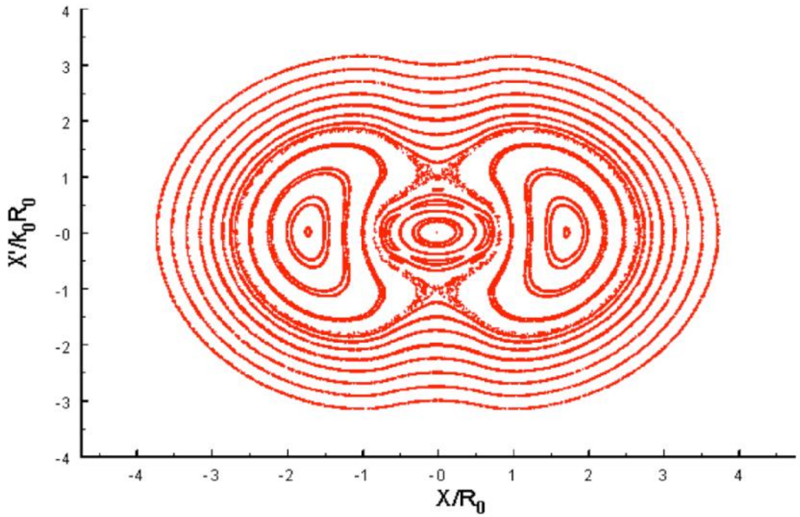
$$\frac{d^2X}{dz^2} + k_0^2 X - F_{\text{sc}} = 0$$

$$F_{\text{sc}} = \begin{cases} KX/R^2, & |X| < R \\ K/X, & |X| \geq R \end{cases}$$



T. Wangler et al. PRSTAB 1998





Betatron Tune Spread

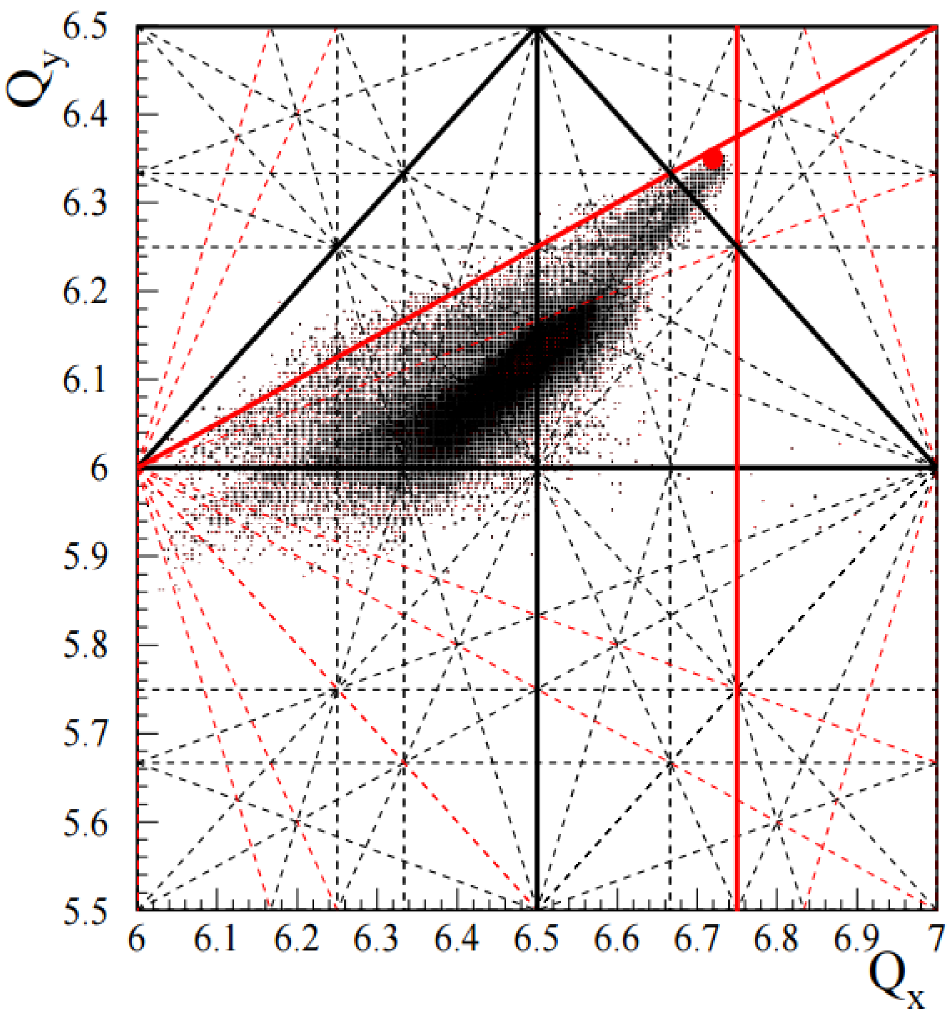
Laslett Tune-shift

$$\boxed{\Delta\nu \approx \frac{N r_0}{2\pi \epsilon_N \beta\gamma^2} F B}$$

Solutions:

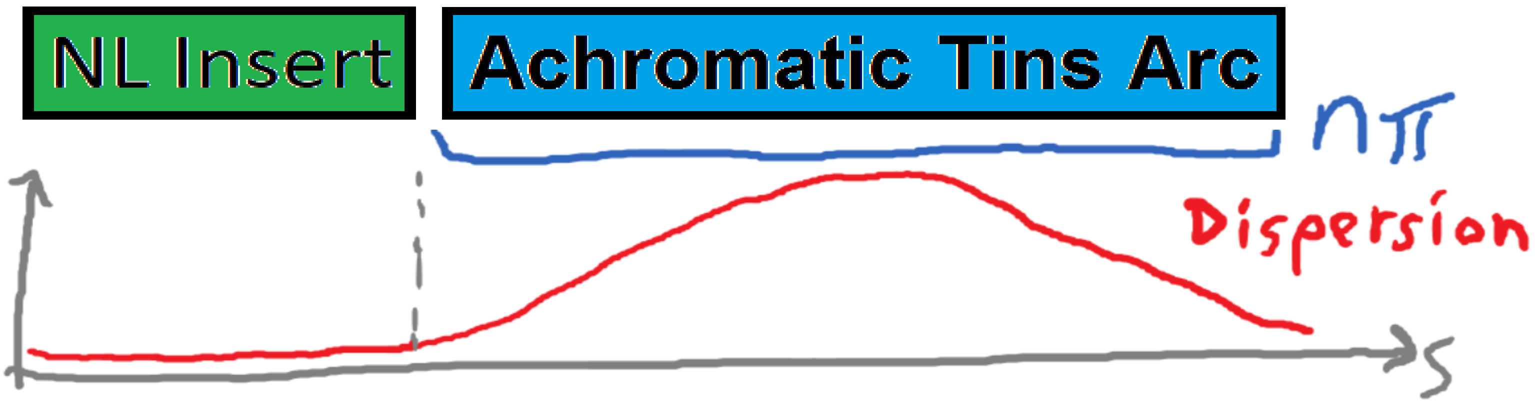
- Increase Injection Energy
- Increase Magnet Aperture
- **Lattice Optics**

Instead of avoiding resonance lines, can be reduce the consequences of crossing them.

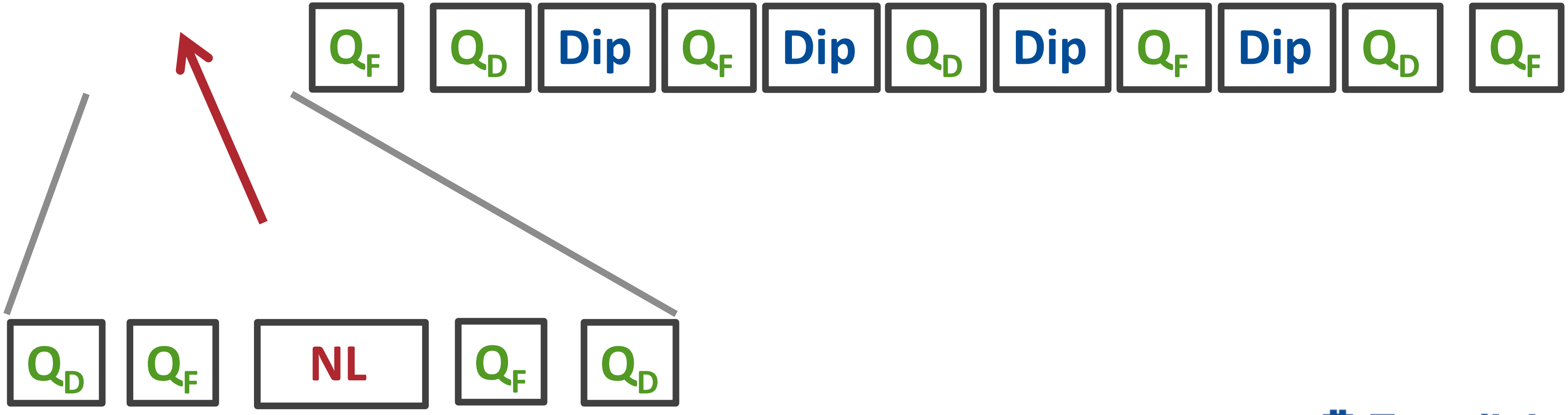


Lattice

Integral Design with Periodicity



Standard FODO Cell:



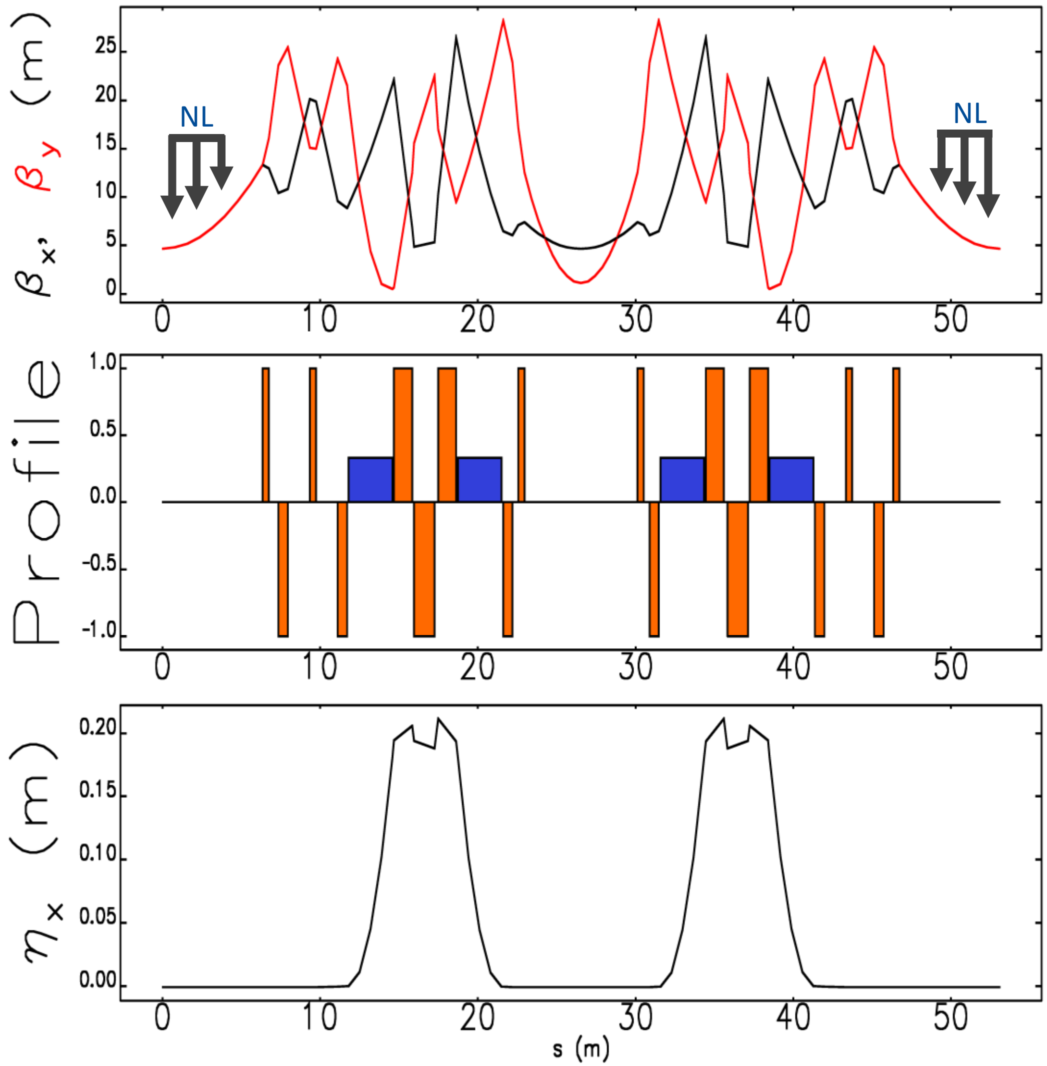
Lattice Requirements

Requirements for DN Integrability:
- Pi-phase advance between nonlinear inserts
- Insertion regions dispersion-free
- Matched beta functions through nonlinear inserts

Further Criteria related to Nonlinear Integrable Optics:
- Large phase-advance through Nonlinear Insertion
- Matched Horizontal and Vertical Chromaticity

Criteria related to RCS Design:
- Low momentum compaction factor.
- Small maximum beta functions
- High lattice superperiodicity
- Compact circumference
- Insertions for RF, inj/ext., collimation, etc.
- Minimize required magnet strengths

Integrable RCS Lattice design



Periodicity: **12**

Circumference: **636 m**

Bend-radius rho: **15.4 m**

Max Beta x,y function: **25 m**

Max Dispersion function: **0.22 m**

RF Insertion length: **7.2 m, 4x 1.3m**

NL Insertion length: **12.7 m**

Insert Phase-Advance: **0.3 x 2π**

Minimum c-value: **3 cm**

Beta at insert center: **5 m**

Betatron Tune: **21.6**

Natural Chromaticity: **-79**

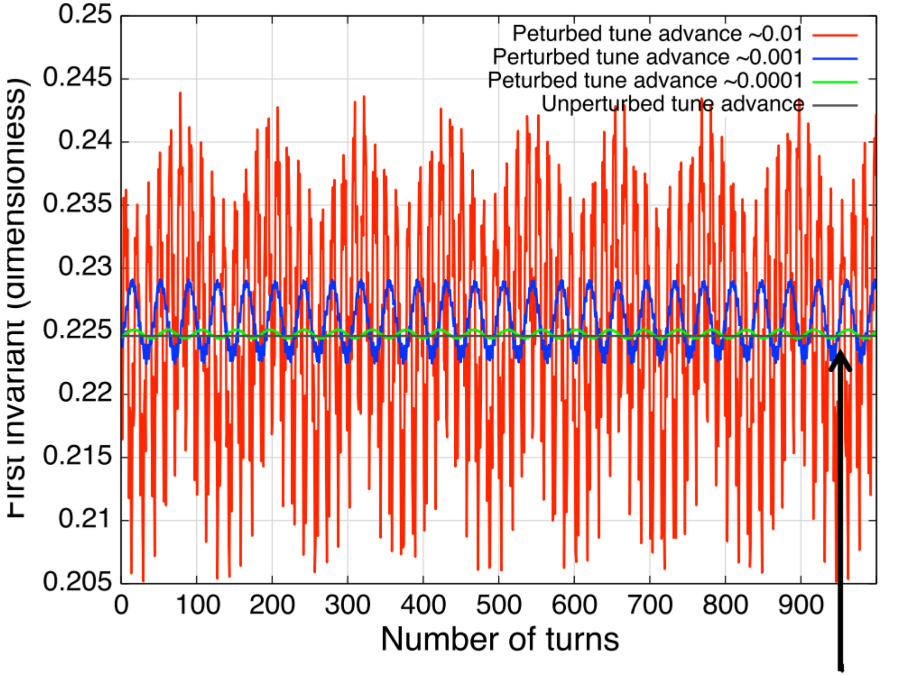
Second-order Chromaticity: **1600**

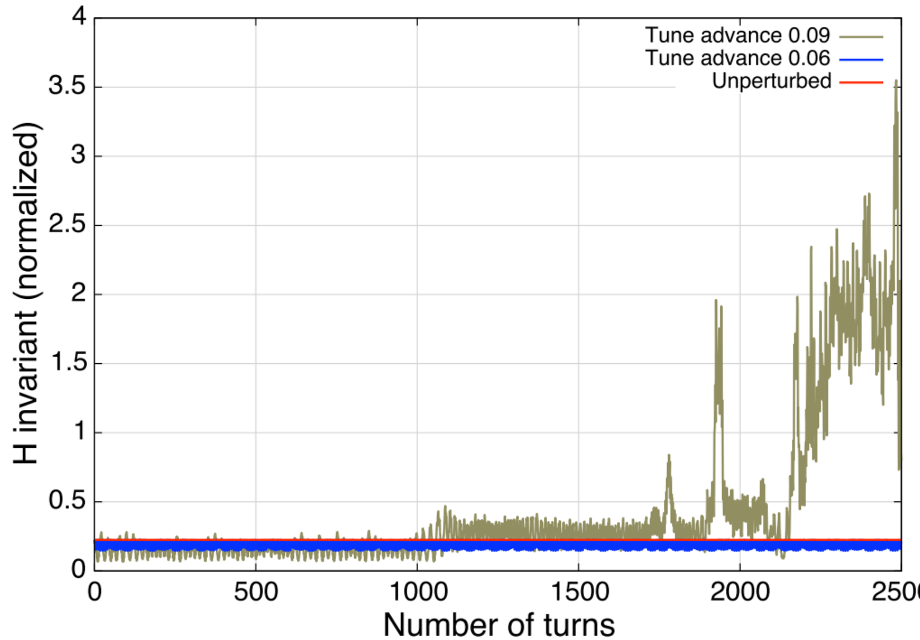
Synchrotron Tune: **0.08**

Perturbative Approach to Phase-errors

Preservation of invariants, single-particle motion, phase-errors

Perturbation of nonlinear t, phase-advance μ, phase error Δφ





unperturbed invariants preserved to within 10^{-7}

Perceptible: ~0.0001

Behaved: ~0.001

Stable: ~0.05

C. Mitchell, LBL

Chromaticity

Chromaticity

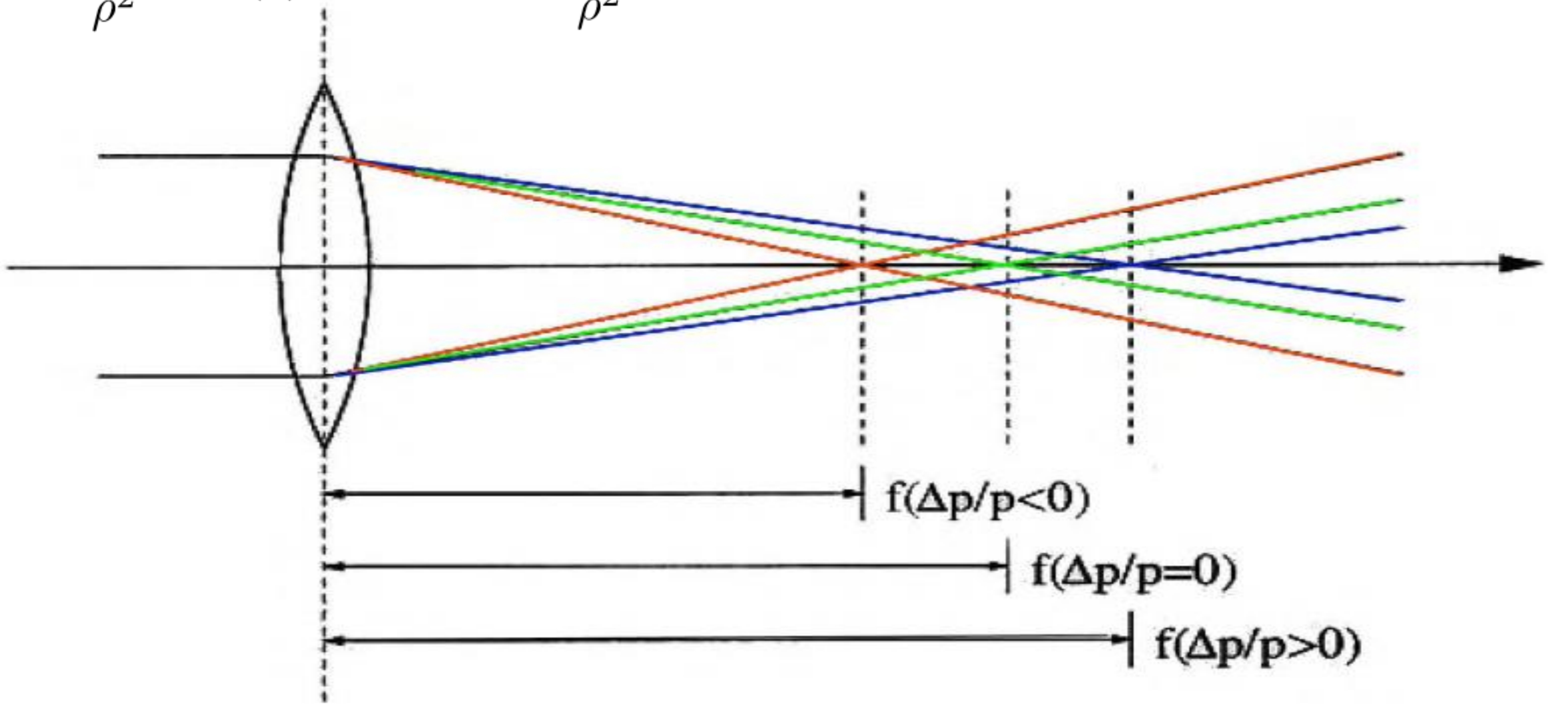
Change in tune with momentum:

$$x''_\beta + (K_x + \Delta K_x \delta) x_\beta = 0$$

$$K_x = \frac{1}{\rho^2} - K(s) \qquad \Delta K_x = -\frac{2}{\rho^2} + K(s)$$

Chromaticity:

$$C_x = \frac{\partial}{\partial \delta} \Delta \nu_x = \frac{1}{4\pi} \int_0^C \beta_x \Delta K_x(s) ds$$



Barletta

Chromatic Shifts

S. Webb: Danilov-Nagaitsev Hamiltonian with chromatic shifts

$$\mathcal{H}_{D-N} = \underbrace{\frac{1}{2}\left(\nu_0 + C_0(\delta)\right)\left(\hat{p}_x^2 + \hat{x}^2 + \hat{p}_y^2 + \hat{y}^2\right) + t\nu_0\mathcal{V}(\hat{x} + D_x(\delta), \hat{y})}_{\mathcal{H}_0} +$$

$$\underbrace{C_\Delta(\delta)\frac{1}{2}\left[\left(\hat{p}_x^2 + \hat{x}^2\right) - \left(\hat{p}_y^2 + \hat{y}^2\right)\right]}_{\Delta\mathcal{H}}$$

$$C_0(\delta) = \frac{C_x + C_y}{2}$$

$$C_\Delta(\delta) = \frac{C_x - C_y}{2}$$

Chromaticity does not have to be zero;

Motion is integrable if horizontal and vertical chromaticity matched.

δ is adiabatic if synchrotron tune (per nonlinear cell) is small.

Natural Chromaticity Matching

Without sextupoles, chromaticity is usually large and negative

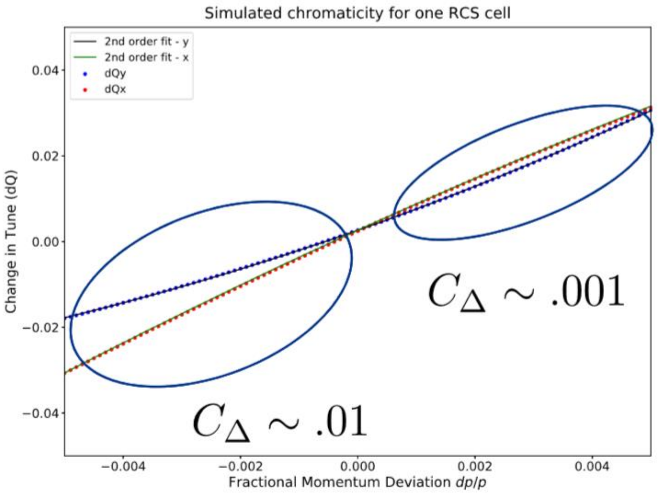
$$C_x = \frac{\partial}{\partial \delta}\Delta\nu_x = \frac{1}{4\pi}\int_0^C \beta_x \Delta K_x(s) ds \qquad \nu_x = \int_0^C \frac{ds}{\beta_x(s)}$$

But horizontal and vertical chromaticity can still be matched by adjusting peaks in beta functions.

Chromaticity is driven by peak betas and betatron tune is driven by minimum betas, so in principle they can be fine-tuned separately.

The nonlinear chromaticity be as similar as possible

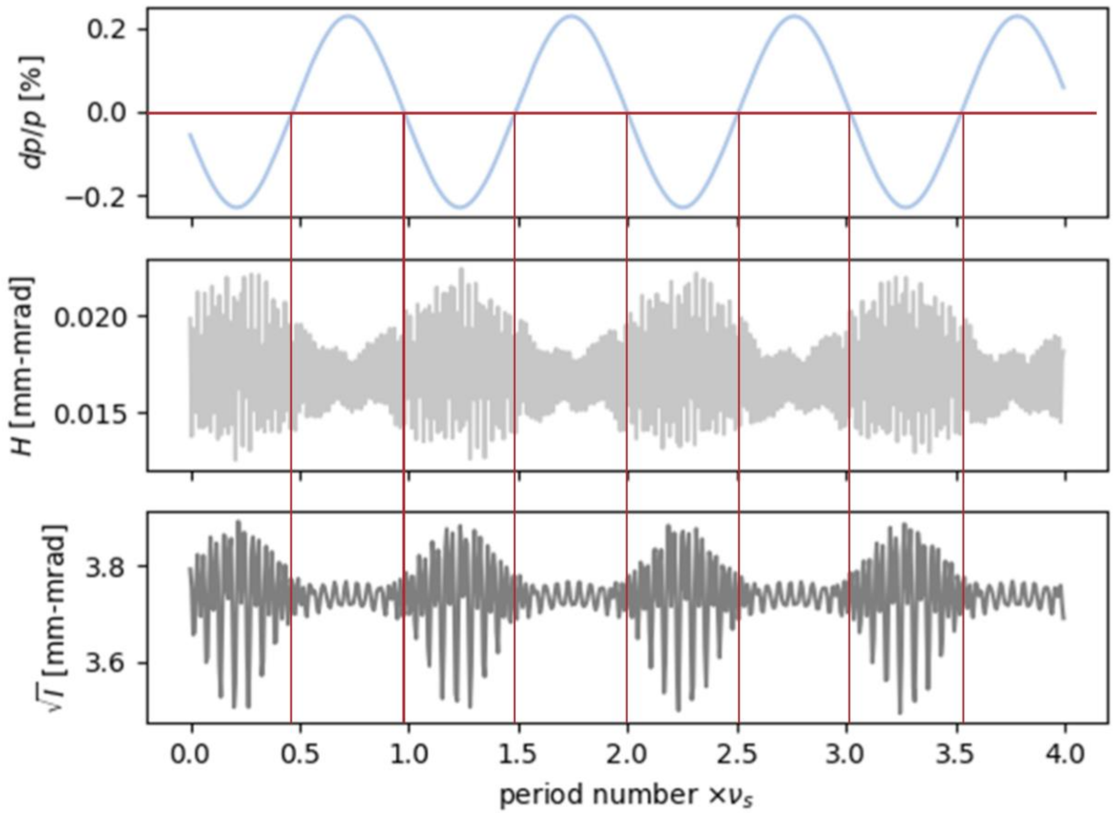
Chromaticity Matching in simulation



chromaticity for an integrable RCS lattice, with regions of almost equal and unequal chromaticities

Parameter	Value
Periodicity	12
Betatron Tune	21.6
Synchrotron Tune	0.08
Phase-advance over insert	0.3 → 2□
Nonlinear Strength t-value	0.3
Elliptic Distance c-value	0.14 $m^{1/2}$

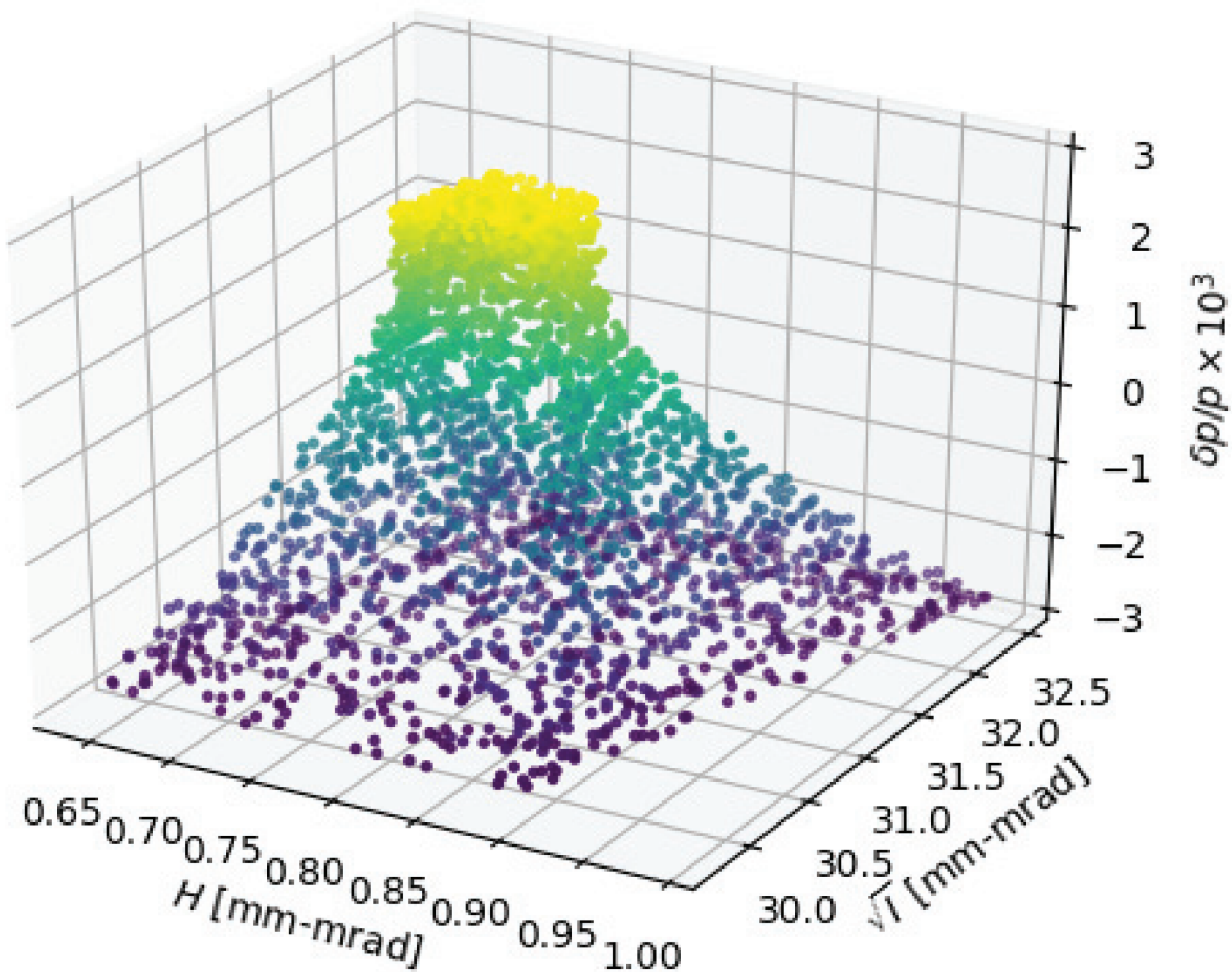
Lattice parameters for an integrable RCS.



variation of the invariants of motion with the synchrotron tune, with worse behavior in the region of unequal chromaticity

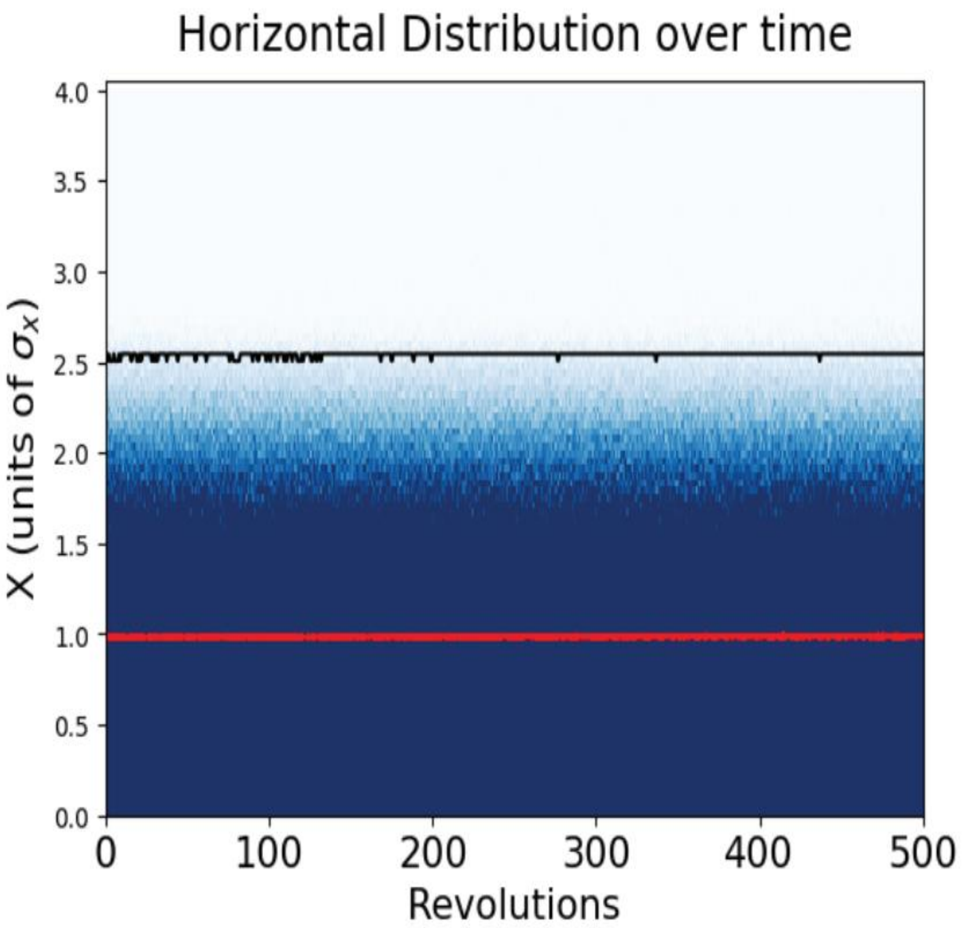
Webb

Chromaticity Matching in simulation



Stable Motion, Massive Chromatic Tune-Spread

Betatron Tune	21.6
Linear Chromaticity	-79
Second-Order Chromaticity	1600
Momentum Compaction	5.9×10^{-4}
Insertion lengths per cell	7.2 m, 4×1.3m
RF Voltage	1.680 MV
Synchrotron Tune	0.08
NL Insertion Length	12.7 m
Phase-advance over insert	0.3
Nonlinear Strength t-value	0.3
Elliptic Distance c-value	$0.14\ \mathrm{m}^{1/2}$
95% Transverse Emittance	20 mm mrad
95% Longitudinal Emittance	$0.09\ \mathrm{eV \cdot s}$
Vertical Lattice Tune Spread	0.52
Horizontal Lattice Tune Spread	0.34
Chromatic Tune spread	0.52



Can we use Sextupoles? Maybe

Sextupoles are the conventional tool for tackling chromaticity:

$$C_x = \frac{1}{4\pi}\int_0^C \beta_x[\Delta K_x(s) + S(s)D(s)]ds$$

Sextupoles create an undesirable nonlinear third-order resonance.

$$G_{3,0,l} \propto \int_0^C \beta_x^{3/2}(s)S(s)e^{j[3\psi_x(s)]}ds \qquad G_{2,\pm1,l} \propto \int_0^C \beta_x^{1/2}(s)\beta_y(s)S(s)e^{j[\psi_x(s)\pm 2\psi_y(s)]}ds$$

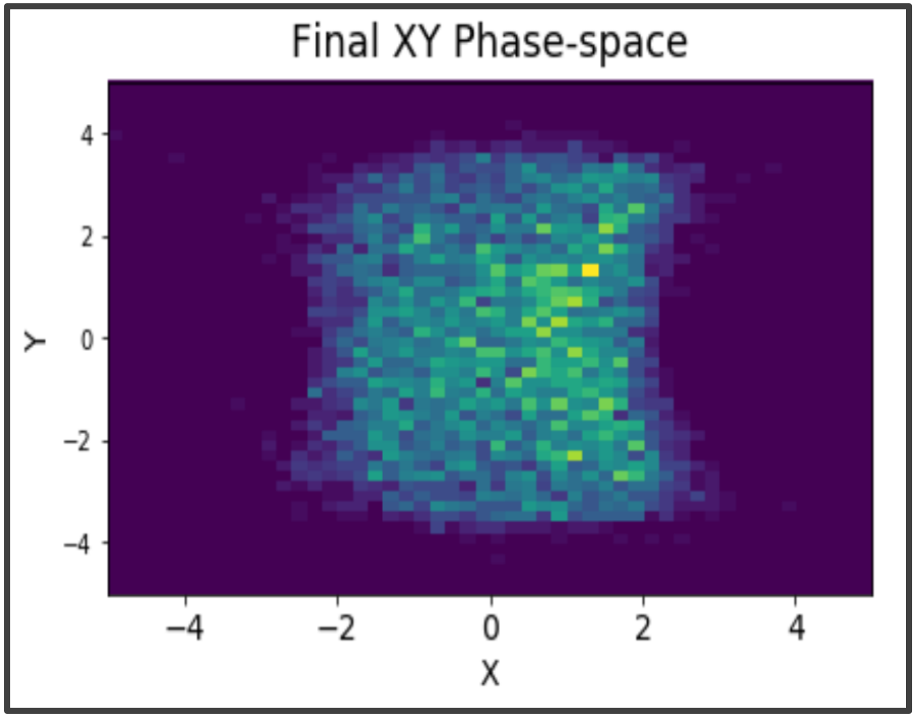
Initial experiments combining strong sextupoles with nonlinear integrable optics were not successful.

Only weak sextupoles are needed to enforce linear chromaticity matching, and no adverse effects observed from this.

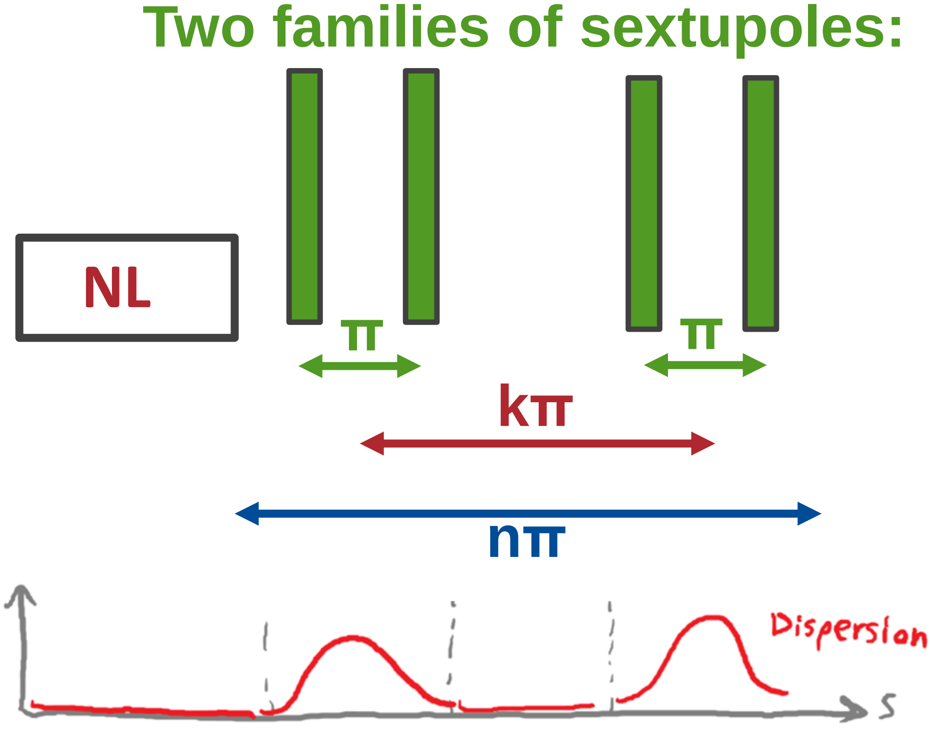
Strong sextupoles should be possible with pi-phase interleaving.

Sextupoles in Phase-space

We can avoid this:



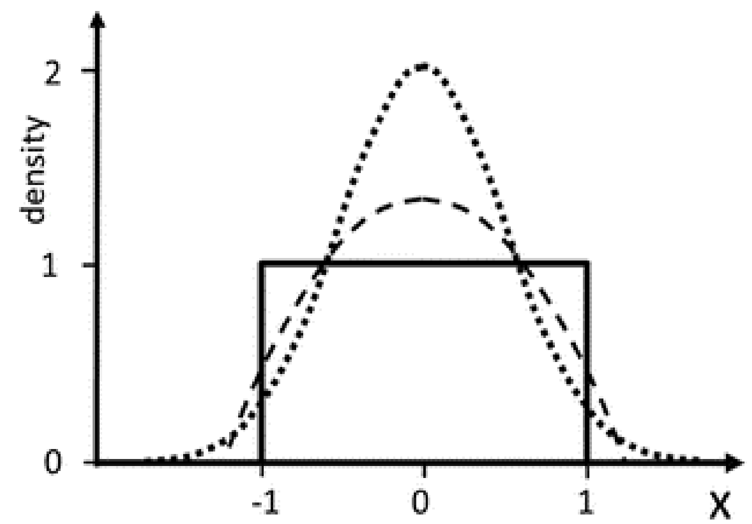
…by doing this:

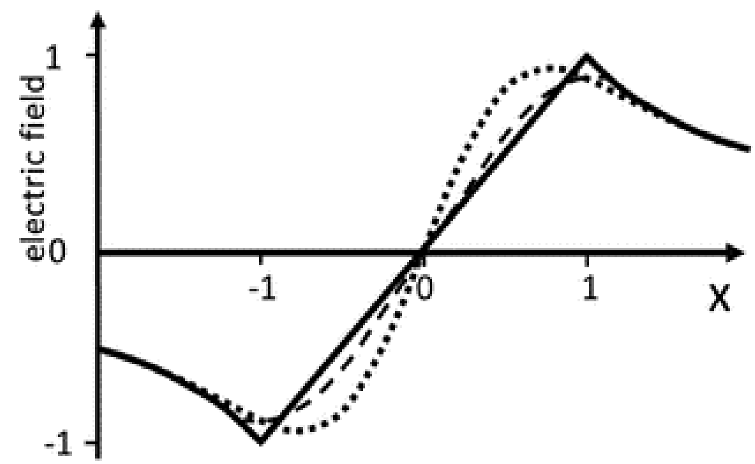


Space-charge

Linear Space-charge Forces

For a KV-beam distribution:
1) All particles have the same H.
2) The 2D projection on the beam is a uniform ellipse.
3) Space-charge forces within the beam are linear.
4) The entire beam will undergo the same tune-shift.





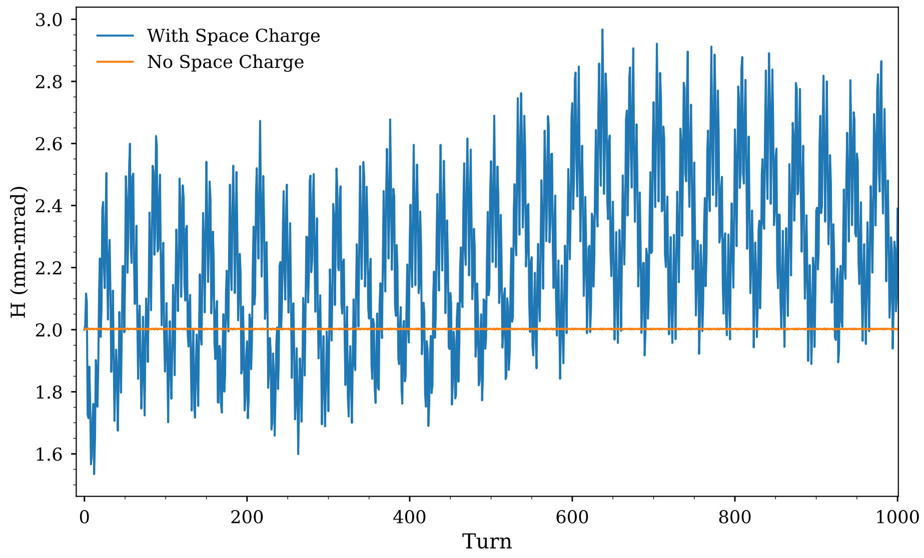
$$\rho_{4D} = \frac{q}{\pi^2 a^2 b^2}\delta\left(1 - \frac{x^2 + p_x^2}{a^2} - \frac{y^2 + p_y^2}{b^2}\right)$$

$$\rho_{2D}(x, y) = \frac{q}{\pi a b}\Theta\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)$$

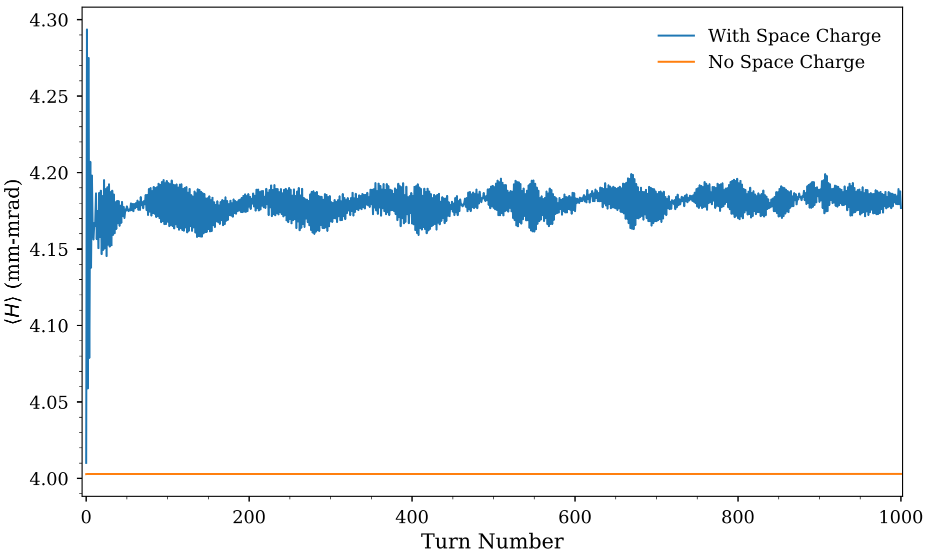
$$\Delta\nu_{sc} = \frac{I(z)}{I_{ave}}\frac{N r_0}{2\pi\epsilon_N \beta\gamma^2}$$

Space-charge breaks integrability

Singe-particle invariants are broken



Ensemble average is better behaved



With space charge:

- ‘Time independence’ of Danilov & Nagaitsev theory is broken
- Both zero-current invariants now fluctuate significantly at 2 frequencies
- Some ensemble properties still appear to be approximately maintained
- Question: Is that enough?

C. Hall

Linear Space-charge Compensation

Rather than use nπ phase-advance between nonlinear inserts, anticipate the space-charge defocusing effect:

$$n\pi + 2\pi\Delta\nu\frac{L}{C} \longrightarrow n\pi$$

This means we will need a separate lattice solution for every operating intensity. But that's what we do.

For a non-KV beam distribution, we can still compensate the linear part of the space-charge defocusing effect in the core of the beam. But in that case, there will be a tune-spread with some phase-error.

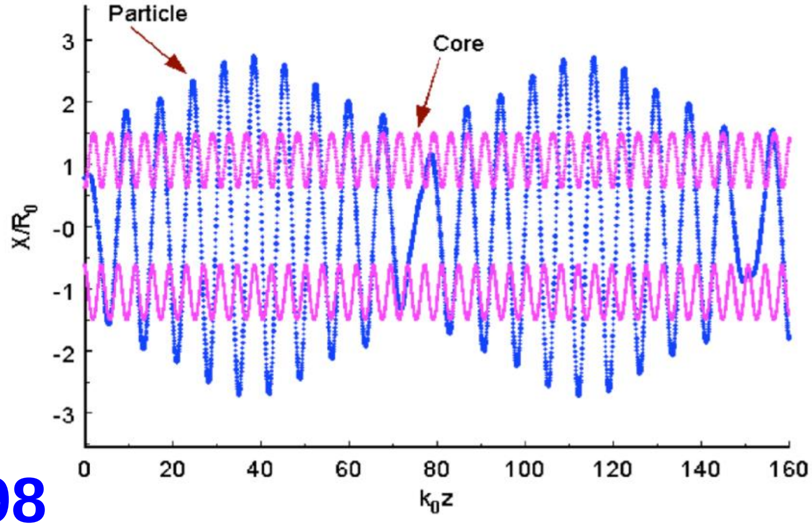
More nonlinear cells help with the space-charge tune-spread, the critical parameter is the space-charge tune-spread per cell.

Performance: Halo from Mismatch Distribution

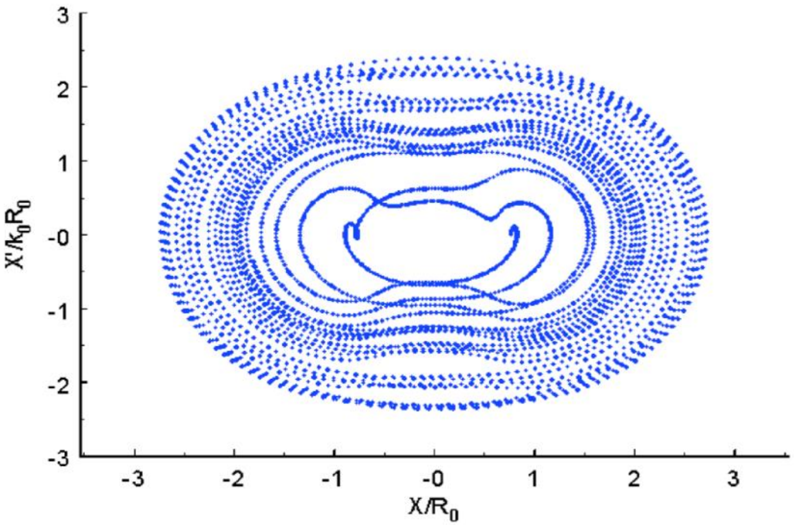
$$\frac{d^2R}{dz^2} + k_0^2 R - \frac{\varepsilon^2}{R^3} - \frac{K}{R} = 0$$

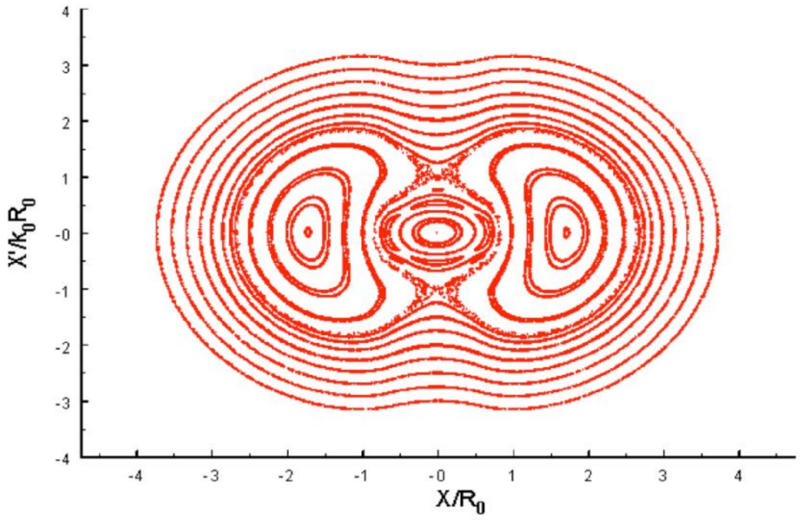
$$\frac{d^2X}{dz^2} + k_0^2 X - F_{\rm sc} = 0$$

$$F_{\rm sc} = \begin{cases} KX/R^2, & |X| < R \\ K/X, & |X| \geq R \end{cases}$$



T. Wangler et al. PRSTAB 1998





Performance: Halo from Mismatch Distribution

Induce a 20% quadrupole mismatch, check for halo.

Can the nonlinear decoherence of the mismatch act to mitigate the formation of the beam halo. Trick: Create pre-halo.

Will we encounter any other difficulty from this strong nonlinearity?

1 Conventional Design, Low Intensity Beam (**dQ = -0.05**)

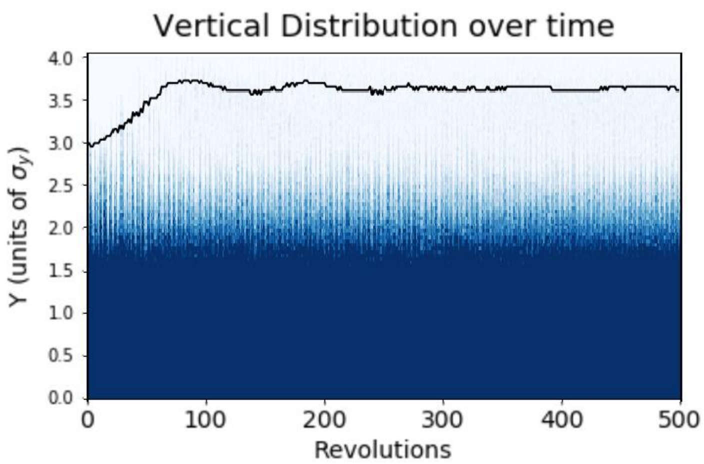
2 Integrable Design, Low Intensity Beam (**dQ = -0.05**)

3 Conventional Design, High Intensity Beam (**dQ = -0.20**)

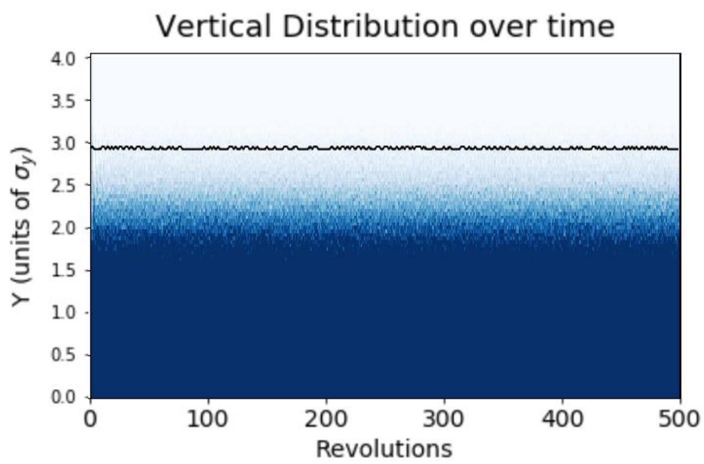
4 Integrable Design , High Intensity Beam (**dQ = -0.20**)

Transverse Beam Halo

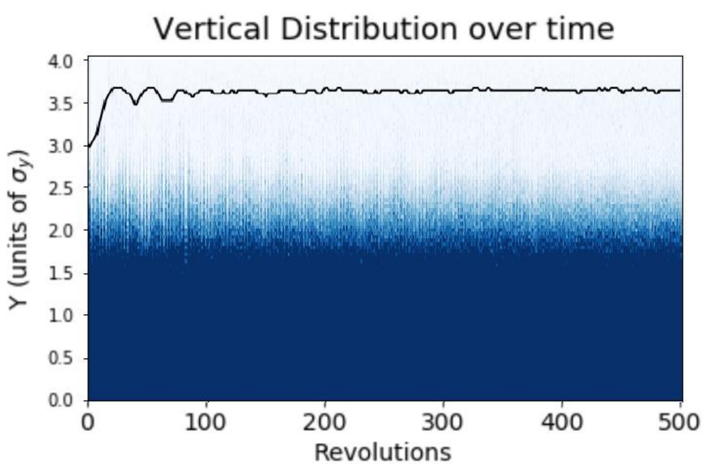
1 Conventional, Low Int.



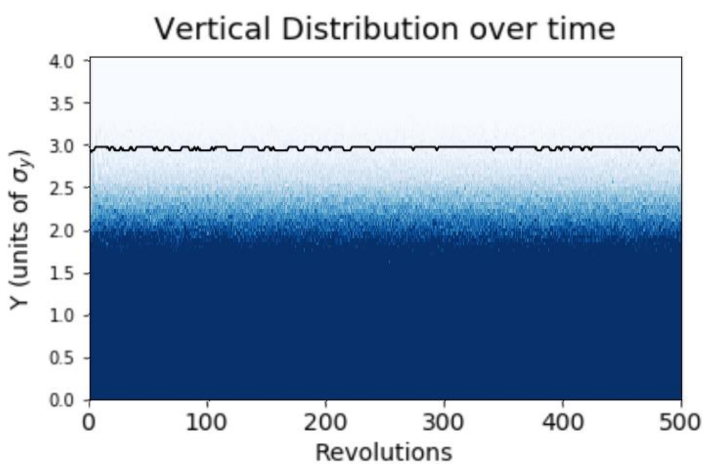
2 Integrable, Low Int.



3 Conventional, High Int.

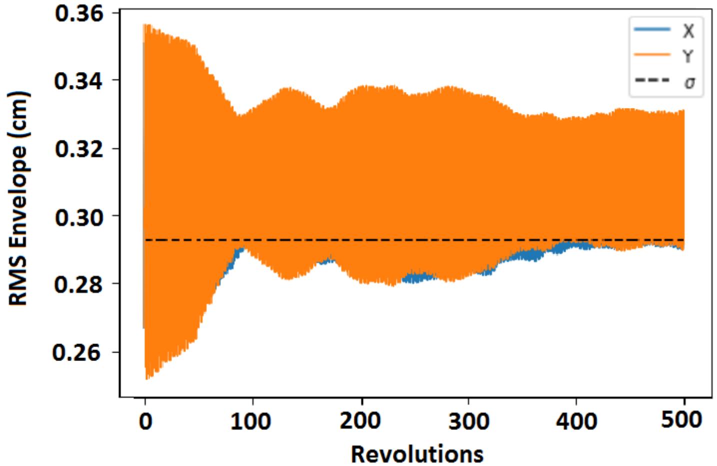


4 Integrable, High Int.

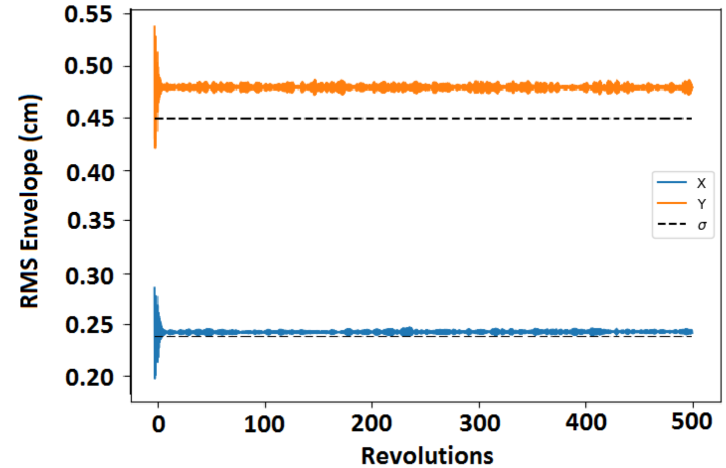


RMS Beam Size

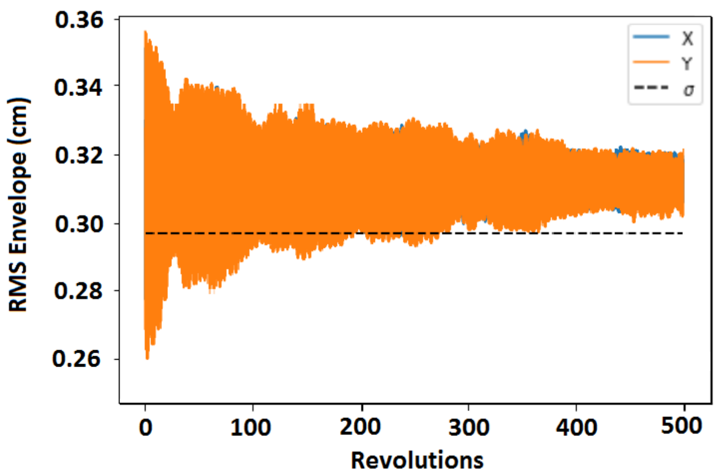
1 Conventional, Low Int.



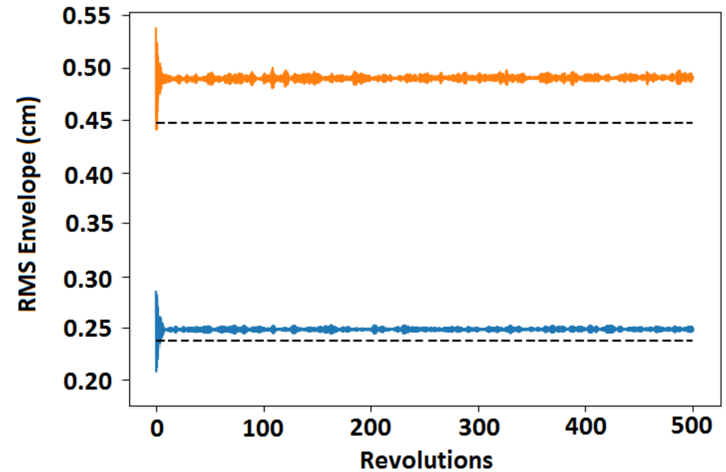
2 Integrable, Low Int.



3 Conventional, High Int.

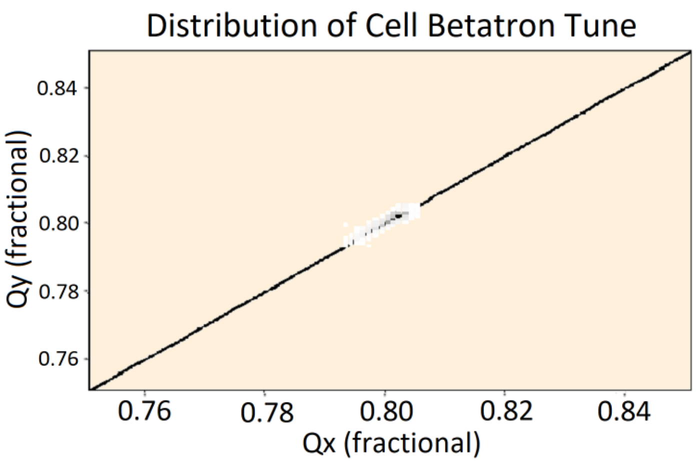


4 Integrable, High Int.

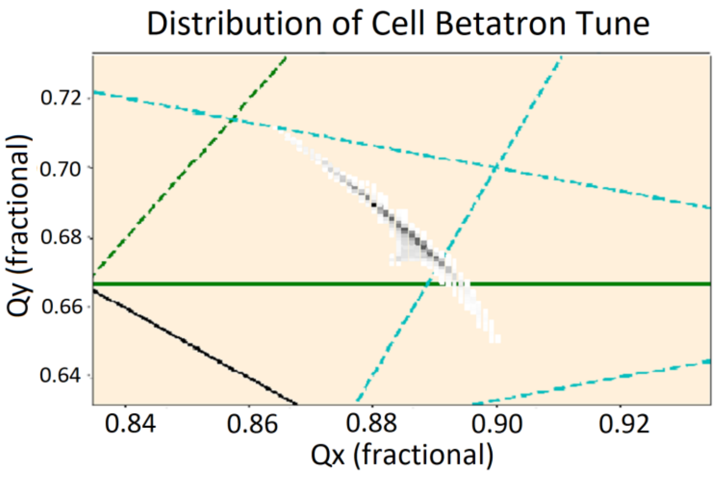


Cell Betatron Tune Distribution

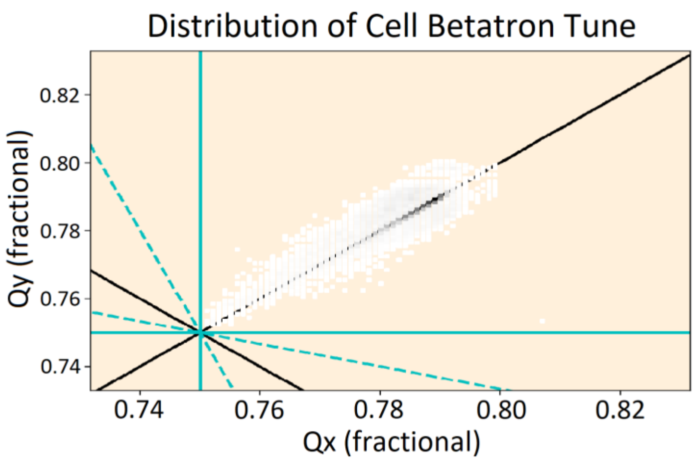
1 Conventional, Low Int.



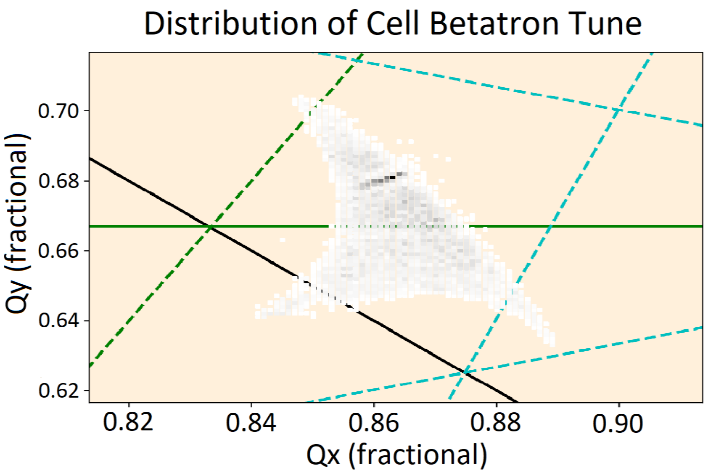
2 Integrable, Low Int.



3 Conventional, High Int.

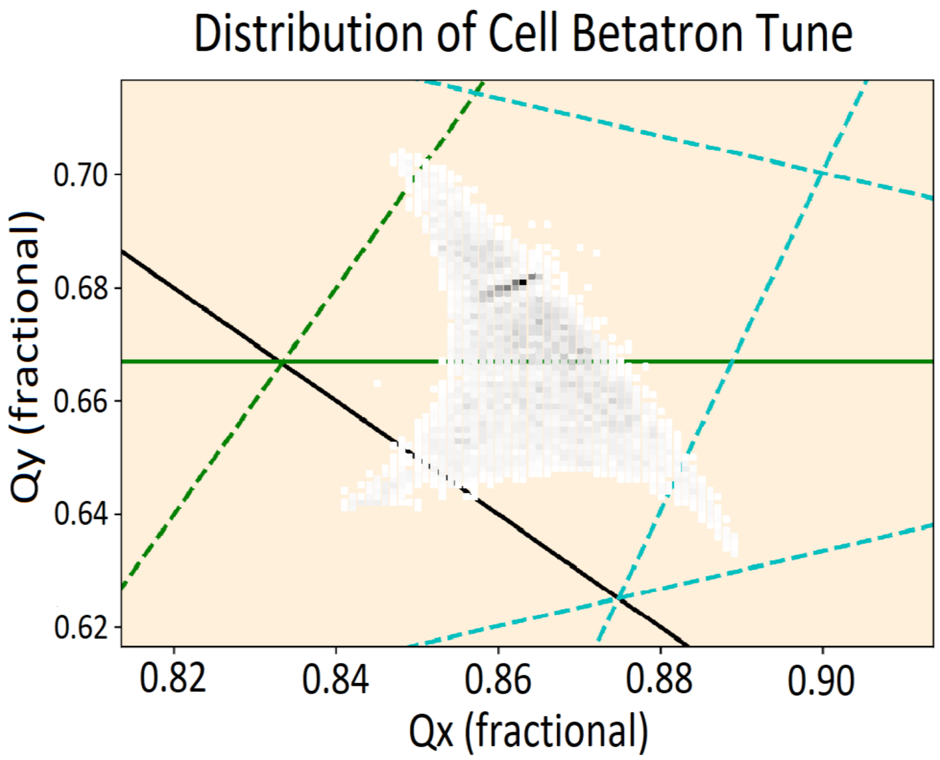


4 Integrable, High Int.

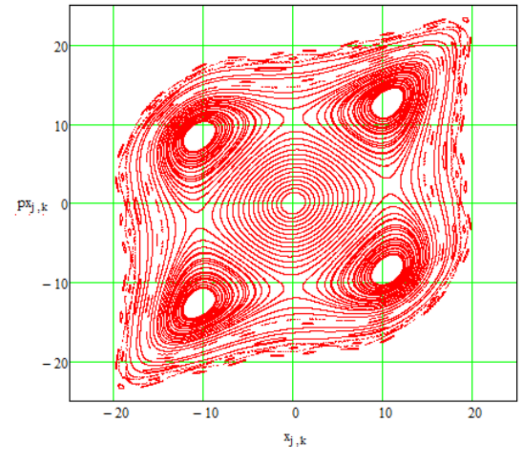


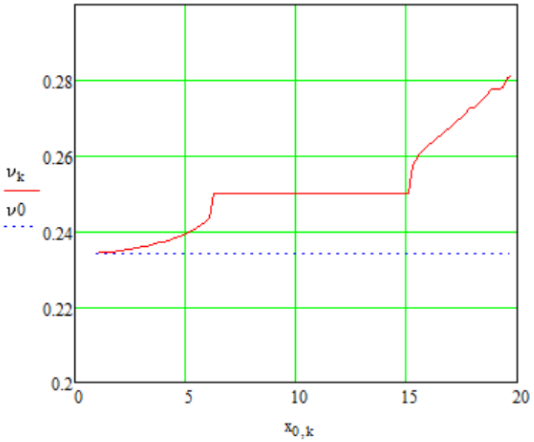
Mode-locking

Periodicity 6, dQ = 0.2



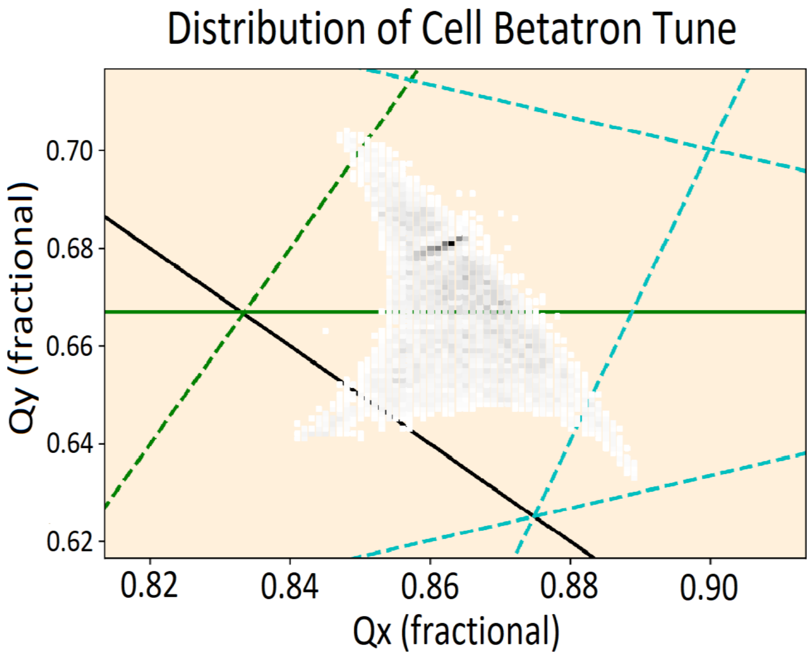
Mode Locking



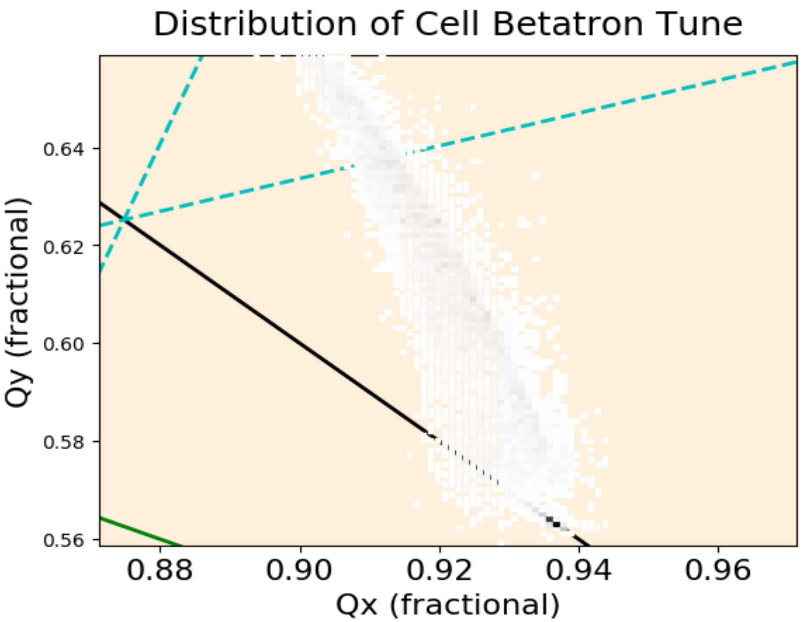


Higher Periodicity -> More Nonlinearity -> Higher Charge

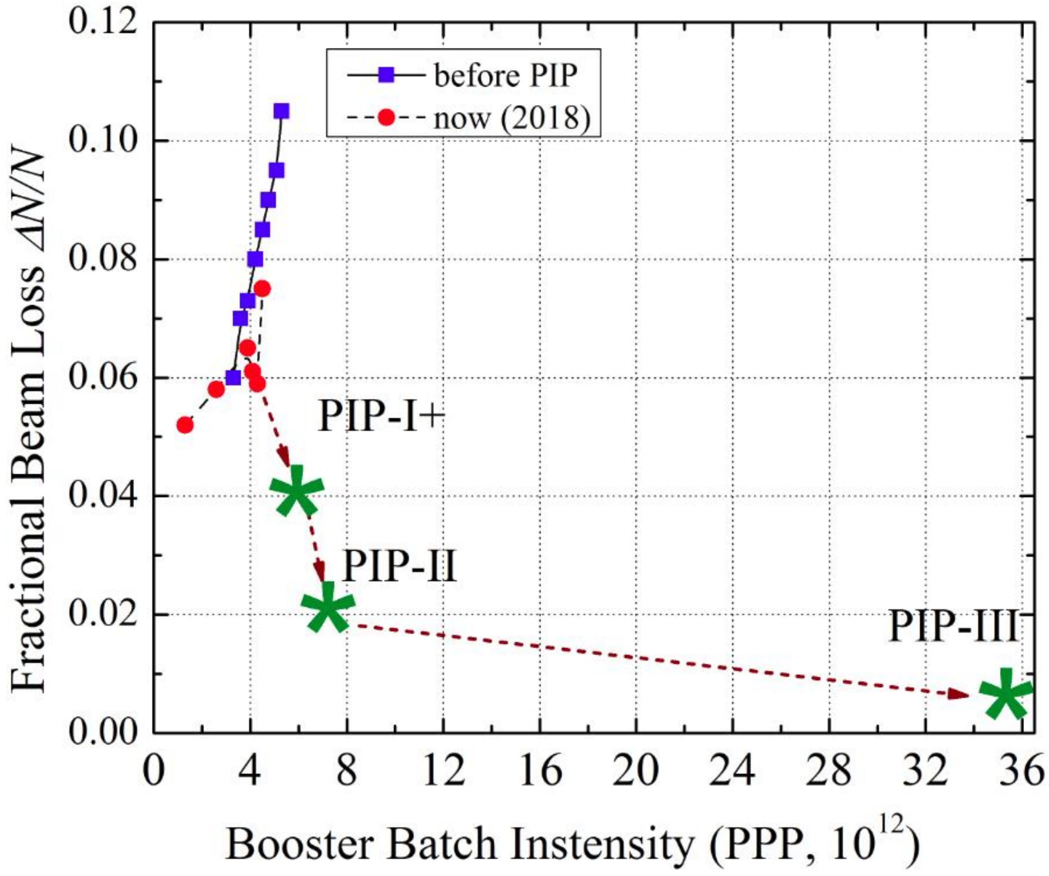
Periodicity 6, dQ = 0.2



Periodicity 12, dQ = 0.4



Fermilab Loss Limits



Radio-activation of particle accelerator a major operation limits. Losses must be kept within absolute limits, which mean power increases require a reduction in *loss rate*.